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An Inverse DEA Model for Two-Stage Systems: A Slack-Based Measure

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Abstract

Conventional Inverse Data Envelopment Analysis (InvDEA) models estimate the input/output levels of Decision-Making Units (DMUs) in order that the efficiency score of units remain unchanged. They ignore slacks in both the efficiency evaluation and the estimation of input/output levels of DMUs, while the existence of slacks can give more accurate information about inputs/outputs of the under-evaluation DMU and estimate inputs/outputs of DMUs more accurate and real. Moreover, the traditional InvDEA models consider DMUs as a black box and do not consider the internal structure of units. To overcome the aforementioned issues, this paper proposes an Inverse Slack-Based Measure of efficiency (InvSBM) model for input estimation in a two-stage network structure. In the model, the efficiency score of under evaluation DMU with a two-stage network character remains unchanged. Since the model is Non-Linear Multi-Objective Programming (NLMOP), a three-stage method is provided in order to estimate input levels of units. Finally, a real case study in the banking industry is presented to show the abilities of the proposed approach.

Keywords: Inverse data envelopment analysis, Two-stage network data envelopment analysis, Slack-based measure of efficiency, Input/output estimation.

1 | Introduction

DEA is a method based on mathematical programming to evaluate the efficiency score of DMUs with multiple inputs and outputs proposed by Charnes et al. [1]. In some cases, the Decision Maker (DM) would like to revise input/output levels with respect to the unchanged efficiency score of units. This is an InvDEA problem which aims to answer the following questions:

I. If, among a group of comparable DMUs, DM wants to increase the output levels of DMUs, how much more input is required in order to fix the efficiency score of units?

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II. If, among a group of homogeneous DMUs, the DM attempts to increase input levels of units, how much more output would be produced such that the efficiency scores of units remain unchanged?

First of all, Wei et al. [2] aim at answering the above questions. Then Yan et al. [3] discussed the InvDEA problem with preference cone constraints. The proposed method allows the DMs to incorporate their preference or important policies over inputs/outputs into the production analysis and resource allocation processes.

Moreover, Jahanshahloo et al. [4] developed the proposed method by Yan et al. [3] and proposed a method to estimate the output levels of a DMU when some or all of its input entities are increased, and its current efficiency level is improved. Then, Jahanshahloo et al. [5] presented an approach to identify extra inputs when the outputs are estimated using the proposed model by Yan et al. [3] and Jahanshahloo et al. [4]. Furthermore, Hadi-Venche and Foroughi [6] suggested a method that takes into account the increase of some inputs/outputs and the decrease due to some of the other inputs/outputs, simultaneously.

Gattoufi et al. [7] suggested a novel application of InvDEA in mergers and acquisitions. They developed a new InvDEA approach to suggest the required level of the inputs and outputs for the merged bank to reach a predetermined efficiency target. Additionally, Amin and Al-Muharrami [8] introduced a new InvDEA approach for mergers with negative data. Amin et al. [9] proposed a method to anticipate whether a merger in a market is generating a major or a minor consolidation. Besides, Shi et al. [10] presented a two-stage cost efficiency DEA model for estimating potential gains from bank mergers.

Since, in the real world, there exist undesirable inputs and outputs in the production process, Jahanshahloo et al. [11] investigated the estimation of inputs/outputs in the presence of undesirable factors. Moreover, Eyni et al. [12] suggested an approach in which InvDEA with preference cone constraints is discussed in a way that in the DMUs, the undesirable inputs and outputs exist simultaneously.

In addition, Ghiyasi [13] developed a theoretical background of the InvDEA with pollution-generating technology that is capable of dealing with undesirable outputs. Then, An et al. [14] proposed two-stage Inverse Data Envelopment Analysis (InvDEA) models with undesirable outputs to formulate resource plans for 16 Chinese-listed commercial banks whose outputs are increased and overall efficiency is kept unchanged in the short term.

In addition, Soleimani-Chamkhorami et al. [15] introduced new models that are based on InvDEA for preserving cost and revenue efficiency when data are changed. Moreover, Soleimani-Chamkhorami et al. [16] proposed a ranking system based on invDEA, which enables the researcher to rank the efficient DMUs appropriately.

Tone [17] proposed a slack-based measure of efficiency in DEA, which deals directly with the input excesses and output shortfalls of the DMU. In contrast to the CCR measure, which is based on the proportional reduction (Enlargement) of input (Output) vectors and which does not take account of slacks, the SBM model deals directly with input excess and output shortfall [17].

The SBM model is a non-radial model that can deal with input excesses and output shortfalls individually. On the other hand, it decreases inputs and outputs, simultaneously. This concept then expanded in many applications of DEA, InvDEA, and network DEA. Hu et al. [18] first showed that neglecting slacks in radial InvDEA could lead to undesirable results, then proposed a revised InvDEA model that can satisfy the requirement of efficiency invariant for DMUs without mix-efficiency.

Since in the existing radial-based InvDEA models, slacks are neglected while evaluating the overall efficiency level of DMUs, in recent years, many InvDEA papers in the non-radial concept have been published. Jahanshahloo et al. [19] studied InvDEA using the non-radial enhanced Russell model. The approach was investigated to identify extra input (Maximum reduction amount)/lack of output (Minimum increase amount) in each of the input/output components.

Mirsalehy et al. [20] introduced a technique that lends its basis to directional SBM for the InvDEA. Furthermore, Zhang and Gui [21] proposed the concept of InvDEA, which is called inverse non-radial DEA. They constructed the mathematical formula of the inverse SBM model, which can overcome the error caused by ignoring slacks. Hu et al. [22] proposed an allocation model based on the SBM model and multi-objective nonlinear programming to find the carbon emissions abatement allocation plan. Moreover, Ghobadi [23] deal with the inverse DEA using the non-radial Enhanced Russell (ER)-measure in the presence of fuzzy data.

It is clear that classical DEA and InvDEA models consider DMUs as a black box and do not consider the internal structure of the units. In contrast, in the real world, most of the DMUs use external inputs and process them in several stages to produce the final outputs of the whole system.

In this paper, we work out with DMUs with two-stage network structures. Two-stage network systems, which consist of two divisions, are connected with intermediate measures. A two-stage network system consumes the exogenous inputs to produce outputs of the first stage, called intermediate products. Then, the intermediate products are used as the inputs of the second stage to produce the output of the second stage, which is also the final output of the whole system

There are many studies on the two-stage network concept. Seiford and Zhu [24] used the independent model to assess the efficiency scores of the first stage, the second stage, and the whole system of 55 U.S. commercial banks. The connections and relationships between stages were not considered in their study. Moreover, Kao and Hwang [25] investigated the efficiency decomposition in a two-stage network system by taking the series relationships of the two sub-processes into account in measuring the efficiencies.

Then, Tone and Tsutsui [26] proposed a network SBM model that can deal with intermediate products formally. Moreover, Lozano [27] presented an SBM model for the general network of processes. The proposed model considers the exogenous inputs and outputs at the system level instead of the process level.

Additionally, Wang et al. [28] provided an examination of the monotonicity of the decomposition weights in a two-stage DEA model with shared resource flows and found that the weight in such a model was not biased towards the second stage. The usage of constant weights in such a model is able to improve the discrimination of the efficient DMUs. Recently, Wang et al. [29] developed a high-tech industrial evaluation framework of technological innovation efficiency based on a two-stage network DEA constructed with shared inputs, additional intermediate inputs, and free intermediate outputs.

Recently, several approaches have been proposed in which the concepts of InvDEA and network DEA are incorporated. For instance, a network-dynamic input-oriented RAM model and its inverse for assessing the sustainability of supply chains were presented by Kalantary et al. [30]. The proposed model changes both inputs and outputs of DMUs so that the efficiency scores of DMUs would remain unchanged.

Furthermore, Kalantary et al. [31] developed a network-dynamic DEA model to evaluate the sustainability of supply chains in multiple periods. Then, they introduced an inverse network DEA model in a dynamic context. In addition, Shiri Daryani et al. [32] employed the input (Output) cost (Price) information and presented the inverse cost (Revenue) efficiency models. They proposed a four-stage method to deal with the InvDEA concept in the two-stage network structures. The method considered the internal structure of DMUs and estimated the input (Output) levels of the units, while the cost (Revenue) scores of all the units remained stable.

Inverse radial models neglect slacks in the input/output estimation procedure since, in some cases, slacks play an essential role in the decision-making process; inverse radial DEA models can not give us more reliable results when the DM aims to estimate input/output levels.

This paper develops a new inverse SBM model in a two-stage network structure to answer this question. If, among a group of comparable DMUs, we increase a certain level of a particular DMU with a two-stage network structure, how many more inputs are required in order that the SBM efficiency score of the units remains unchanged? To overcome this issue, we propose an inverse two-stage SBM model. Since the proposed model is an NLMOP, a three-stage method is presented to estimate the input levels of the DMUs.

The remainder of this paper is organized as follows: In Section 2, the concepts of InvDEA, two-stage network DEA, and slack-based measure of efficiency are reviewed. The inverse two-stage SBM model and a three-stage method to solve the model have been provided in Section 3. In Section 4, to examine the validity of the proposed model, a case study is presented.

2 | Preliminary

In this section, the concepts of InvDEA, two-stage network DEA, and slack-based measures of efficiency are reviewed.

2.1 | Inverse Data Envelopment Analysis

Inverse DEA which was proposed by Wei et al. [2] to answer the following questions:

I. If, among a group of homogeneous DMUs, the input levels of DMU_o increases from x_o to $\alpha_o = x_o + \Delta x_o \left(\Delta x_o \geq 0\ ,\ \Delta x_o \neq 0\right)$, how much more output $\beta_o = y_o + \Delta y_o$ would be produced, in order for the efficiency score of $DMU_o\left(\theta_o^*\right)$ remains unchanged?

II. If, among a group of comparable DMUs, the output levels of DMU_o increases from y_o to $\beta_o = y_o + \Delta y_o \left(\Delta y_o \geq 0\ ,\ \Delta y_o \neq 0\right)$, how much input $x_o = x_o + \Delta x_o$ is required in order for unchanged the efficiency score of $DMU_o\left(\theta_o^*\right)$?

To solve the mentioned problems, let us assume that the output levels of DMU_o are changed from y_o to $\beta_{ro} = y_{ro} + \Delta y_{ro} \left(\Delta y_{ro} \geq 0\ ,\ \Delta y_{ro} \neq 0\ ,\ r = 1,...,s\right)$. We need to estimate the input levels $\alpha_i = x_i + \Delta x_i \left(i = 1,...,m\right)$ of DMU_o such that the efficiency score of DMU_o would still be θ_o^*. The InvDEA model is as follows:

$$\begin{aligned}
&\min\ \left(\alpha_1,\alpha_2,....,\alpha_m\right),\\
&\text{s.t.}\ \sum_{i=1}^{m}\lambda_j x_{ij} \leq \theta_o^*\alpha_i,\ i=1,...,m,\\
&\sum_{r=1}^{s}\lambda_j y_{ij} \geq \beta_{ro},\ r=1,...,s,\\
&\lambda_j \geq 0,\ j=1,...,n,\\
&\alpha_i \geq 0,\ i=1,...,m.
\end{aligned} \tag{1}$$

where all $x_{ij}\left(i=1,...,m\right)$, $y_{rj}\left(r=1,...,s\right)$ and $\beta_{ro}\left(r=1,...,s\right)$ are given, and we attempt to obtain $\alpha_i\left(i=1,...,m\right)$s. It is clear that *Model (1)* is a MOLP.

Definition 1 (Weak efficient solution). Assume that $\left(\lambda,\alpha\right)$ is a feasible solution of *Model (1)*. If there is no feasible solution $\left(\bar{\lambda},\bar{\alpha}\right)$ such that for all $i=1,...,m$, $\bar{\alpha_i} \leq \alpha_i$, then we say that $\left(\lambda,\alpha\right)$ is a weak efficient solution of *Model (1)*.

It has been proven by Ghiyasi [33] that if a revision for DMU_o from $\left(x_o,y_o\right)$ to $\left(\alpha_o,\beta_o\right)$ is considered and it is assumed that $\left(\lambda_o,\alpha_o\right)$ is a weak efficient solution of *Model (1)*, then the efficiency scores of all DMUs would stay unchanged after revision.

There are several methods to solve the MOLP *Model (1)*. Assume all inputs are weighed (Priced), and the weights (Values) are known. Let $w_i \left(i = 1,...,m\right)$ be the value weight for i[th] input. To solve *Model (1)*, the weighted sum method is considered:

$$
\begin{aligned}
&\min \sum_{i=1}^{m} w_i \alpha_i, \\
&\text{s.t.} \quad \sum_{i=1}^{m} \lambda_j x_{ij} \le \theta_o^* \alpha_i, \ \ i = 1,...,m, \\
&\qquad \sum_{r=1}^{s} \lambda_j y_{ij} \ge \beta_{ro}, \ \ r = 1,...,s, \\
&\qquad \lambda_j \ge 0, \ \ j = 1,...,n, \\
&\qquad \alpha_i \ge 0, \ \ i = 1,...,m.
\end{aligned} \tag{2}
$$

2.3 | Slack-Based Measure of Efficiency

The radial DEA models neglect slacks in the assessment of the efficiency scores of units. In contrast to the radial models, non-radial DEA models can overcome the error caused by ignoring slacks and provide more valuable information about inputs and outputs for decision-making by considering slacks. Since the radial models ignore slacks in the evaluation of the efficiency scores of units, Tone [17] proposed a slack-based measure of efficiency, which is called the SBM model.

SBM deals directly with the input excesses and the output shortfalls of the under-evaluation DMU, which are called slacks. It has three crucial features. First, it is invariant with respect to the units of data (Unit invariant). Second, it is monotone decreasing in each slack (Monotone). Third, it is reference-set dependent, which means the slack-based measure should be determined only by consulting the reference-set of the DMU concerned. Let the vectors $s^- \in \mathbb{R}^m$ and $s^+ \in \mathbb{R}^s$ indicate the input excesses and the output shortfalls, respectively. Using these expressions, the SBM model proposed by Tone [17] is as follows:

$$
\begin{aligned}
&\rho_o^* = \min \rho_o = \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}, \\
&\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \qquad i = 1,...,m, \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \qquad r = 1,...,s, \\
&\qquad \lambda_j \ge 0, \qquad j = 1,...,n, \\
&\qquad s_i^- \ge 0, \qquad i = 1,...,m, \\
&\qquad s_r^+ \ge 0, \qquad r = 1,...,s.
\end{aligned} \tag{3}
$$

SBM is a fractional program, which can be transformed to the linear form by Charnes-Cooper [34] transformation. Assume $\left(\lambda^*, s^{-^*}, s^{+^*}\right)$ is the optimal solution of *Model (3)*.

Definition 3 (SBM-efficient). DMU_o is SBM-efficient, if $\rho_o^* = 1$. In other words, DMU_o is SBM-efficient if $s_i^{-*} = 0$, $(i = 1,...,m)$ and $s_r^{+*} = 0$, $(r = 1,...,s)$.

2.4 | Non-Radial Two-Stage Network Data Envelopment Analysis

In a network with a two-stage structure, the first stage consumes external inputs $x_{ij}(i = 1,...,m)$ and produces $z_{gj}(g = 1,...,h)$. Then $z_{gj}(g = 1,...,h)$ is consumed by the second stage to produce $y_{rj}(r = 1,...,s)$ as the outputs, which are simultaneously the final outputs of the whole system. $z_{gj}(g = 1,...,h)$ which is the output of the first stage, and the inputs of the second stage are called the intermediate products of the entire system. The structure of a basic two-stage network is depicted in *Fig. 1*.

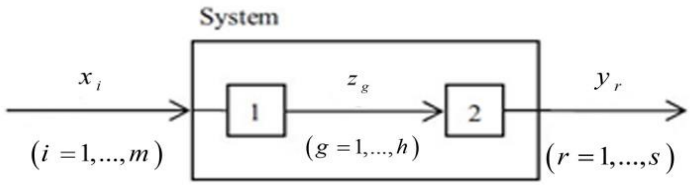

Fig. 1. Structure of the basic two-stage network.

Lozano [27] presented a model to evaluate the efficiency score of a general network of processes. The special case of the proposed model for evaluating the efficiency score of $DMU_o = (x_o, z_o, y_o)$ with a two-stage network structure is:

$$
\begin{aligned}
E_o = \min \ & \frac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_r^+}{y_{ro}}}, && \\
\text{s.t.} \ & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, && i = 1,...,m, \\
& \sum_{j=1}^{n} \mu_j y_{rj} - s_r^+ = y_{ro}, && r = 1,...,s, \\
& \sum_{j=1}^{n} (\lambda_j - \mu_j) z_{gj} \geq 0, && g = 1,...,h, \\
& \lambda_j \geq 0, && j = 1,...,n, \\
& \mu_j \geq 0, && j = 1,...,n, \\
& s_i^- \geq 0, && i = 1,...,m, \\
& s_r^+ \geq 0, && r = 1,...,s.
\end{aligned}
\quad (4)
$$

The input-oriented version of *Model (4)* is:

$$
\begin{aligned}
& E_o^{input^*} = \min \ 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}, \\
& \text{s.t.} \quad \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = x_{io}, \qquad i = 1,\dots,m, \\
& \qquad \sum_{j=1}^{n}\mu_j y_{rj} \ge y_{ro}, \qquad r = 1,\dots,s, \\
& \quad \sum_{j=1}^{n}\left(\lambda_j - \mu_j\right) z_{gj} \ge 0, \qquad g = 1,\dots,h, \\
& \qquad \lambda_j \ge 0, \qquad j = 1,\dots,n, \\
& \qquad \mu_j \ge 0, \qquad j = 1,\dots,n, \\
& \qquad s_i^- \ge 0, \qquad i = 1,\dots,m.
\end{aligned} \tag{5}
$$

Assume that $\left(E_o^{input^*}, \lambda^*, \mu^*, s^{-^*}\right)$ is an optimal solution of the above model.

3 | Inverse Two-Stage Network Slack-Based Measure

Conventional InvDEA models estimate inputs/outputs of DMUs without considering slacks, while the existence of slacks can.

I. Give more accurate information about inputs/outputs of under evaluation DMU.

II. Estimate inputs/outputs of DMUs more accurate and real.

Moreover, the aforementioned models in the literature review consider DMUs as a black box and ignore the internal structure of the units. Since the non-radial DEA evaluates the efficiency score of DMU concerned by considering slacks, in this paper, we establish an InvDEA model based on non-radial DEA, which involves both input excesses and output shortfalls simultaneously. Hence, in this section, we incorporate InvDEA and network DEA concepts and propose the inverse two-stage network SBM in which the input and output slacks are considered in the process of input estimation. Now consider *Model (5)* and assume that the output levels of $DMU_o = \left(x_o, z_o, y_o\right)$ increases from y_o to $\beta_o = y_o + \Delta y_o$, we need to estimate the input vector $\alpha_o = x_o + \Delta x_o$ in order that the efficiency score of DMU_o would still be $E_o^{input^*}$ that obtained from *Model (5)*. The inverse two-stage network SBM model is:

$$
\begin{aligned}
& \text{Min} \quad \left(\alpha_{1o}, \alpha_{2o}, \dots, \alpha_{mo}\right), \\
& \text{s.t.} \quad \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \alpha_{io}, \qquad i = 1,\dots,m, \\
& \qquad \sum_{j=1}^{n}\mu_j y_{rj} \ge \beta_{ro}, \qquad r = 1,\dots,s, \\
& \quad \sum_{j=1}^{n}\left(\lambda_j - \mu_j\right) z_{gj} \ge 0, \qquad g = 1,\dots,h, \\
& \quad E_o^{input^*} = 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^{-*}}{\alpha_{io}}, \\
& \qquad \lambda_j \ge 0, \qquad j = 1,\dots,n, \\
& \qquad \mu_j \ge 0, \qquad j = 1,\dots,n, \\
& \qquad s_i^- \ge 0, \qquad i = 1,\dots,m, \\
& \qquad \alpha_{io} \ge 0, \qquad i = 1,\dots,m,
\end{aligned} \tag{6}
$$

Theorem 1. Suppose that the input-oriented SBM efficiency score of DMU_o under *Model (5)* is $E_o^{input^*}$ and the outputs of DMU_o are going to increase from y_o to $\beta_o = y_o + \Delta y_o \left(\Delta y_o \geq 0\ ,\ \Delta y_o \neq 0\right)$.

I. Let $\left(\overline{\alpha_o}, \overline{\lambda}, \overline{\mu}, \overline{s^-}\right)$ be a Pareto solution of *Model (6)*. Then, the efficiency score of DMU_o under new inputs-outputs $\left(\alpha_o, \beta_o\right)$ remains unchanged.

Proof: consider the following model, which evaluates the efficiency score of a new DMU_o, which is called $DMU_{n+1} = \left(\alpha_o^*, z_o, \beta_o\right)$.

$$
\begin{aligned}
\theta_o^* = \min\ & 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}} \\
\text{s.t.}\ & \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \alpha_{io}, && i = 1,\ldots,m, \\
& \sum_{j=1}^{n}\mu_j y_{rj} \geq \beta_{ro}, && r = 1,\ldots,s, \\
& \sum_{j=1}^{n}\left(\lambda_j - \mu_j\right) z_{gj} \geq 0, && g = 1,\ldots,h, \\
& \lambda_j \geq 0, && j = 1,\ldots,n, \\
& \mu_j \geq 0, && j = 1,\ldots,n, \\
& s_i^- \geq 0, && i = 1,\ldots,m.
\end{aligned}
\qquad (7)
$$

We are going to prove that $\theta_o^* = E_o^{input^*}$. Let $\left(\overline{\alpha_o}, \overline{\lambda}, \overline{\mu}, \overline{s^-}\right)$ be a Pareto solution of *Model (6)*. Since the constraints

$$
\begin{aligned}
& \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- = \alpha_{io}, && i = 1,\ldots,m, \\
& \sum_{j=1}^{n}\mu_j y_{rj} \geq \beta_{ro}, && r = 1,\ldots,s, \\
& \sum_{j=1}^{n}\left(\lambda_j - \mu_j\right) z_{gj} \geq 0, && g = 1,\ldots,h, \\
& \lambda_j \geq 0, && j = 1,\ldots,n, \\
& \mu_j \geq 0, && j = 1,\ldots,n, \\
& s_i^- \geq 0, && i = 1,\ldots,m,
\end{aligned}
$$

The same, it is clear that $\left(\overline{\alpha_o}, \overline{\lambda}, \overline{\mu}, \overline{s^-}\right)$ is a feasible solution of *Model (8)*, then $\theta_o^* \geq E_o^{input^*}$, where θ_o^* is the optimal value of *Model (7)*. Now, by contraction $\theta_o^* > E_o^{input^*}$, we can obtain some new inputs $\hat{\alpha}_o = \overline{\alpha}_o + \Delta\overline{\alpha}_o \left(\Delta\overline{\alpha}_o \geq 0\ ,\ \Delta\overline{\alpha}_o \neq 0\right)$ such that in *Model (7)* $\text{Eff}\left(\hat{\alpha}_o, z_o, \beta_o\right) = \theta_o^*$, since $\theta_o^* > E_o^{input^*}$. At the same time, we get the parameters $\left(\hat{\lambda}, \hat{\mu}, \hat{s}^-\right)$ in model (6). To conclude the proof, we claim that $\left(\hat{\alpha}_o, \hat{\lambda}, \hat{\mu}, \hat{s}^-\right)$ is a feasible solution of *Model (6)*. Because $\text{Eff}\left(\hat{\alpha}_o, z_o, \beta_o\right) = E_o^{input^*} = \text{Eff}\left(x_o, z_o, y_o\right)$, the constraint $E_o^{input^*} = 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^{-*}}{\alpha_{io}}$ is satisfied and the remaining constraints are identical, $\left(\hat{\alpha}_o, \hat{\lambda}, \hat{\mu}, \hat{s}^-\right)$ is a feasible solution of *Model (6)*. Because

$\hat{\alpha}_o \le \bar{\alpha}_o$, $\hat{\alpha}_o \ne \bar{\alpha}_o$, this contradicts the fact that $\left(\overline{\alpha_o}, \bar{\lambda}, \bar{\mu}, \overline{s^-}\right)$ is a Pareto solution. Therefore, the assumption $\theta_o^* > E_o^{input^*}$ is wrong, which means $\theta_o^* > E_o^{input^*}$.

II. Conversely, let $\left(\overline{\alpha_o}, \bar{\lambda}, \bar{\mu}, \overline{s^-}\right)$ be a feasible solution of *Model (7)*. If the efficiency score $\left(\overline{\alpha_o}, z_o, \beta_o\right)$ under *Model (5)* is still $E_o^{input^*}$, then $\left(\overline{\alpha_o}, \bar{\lambda}, \bar{\mu}, \overline{s^-}\right)$ must be a Pareto solution of *Model (6)*.

Proof: Let $\left(\overline{\alpha_o}, \bar{\lambda}, \bar{\mu}, \overline{s^-}\right)$ be a feasible solution of *Model (6)* and $\overline{\theta_o}$ be the efficiency score of $\left(\overline{\alpha_o}, z_o, \beta_o\right)$ under *Model (6)*. If $\left(\overline{\alpha_o}, z_o, \beta_o\right)$ is not a Pareto solution of *Model (6)*, we can claim that $\bar{\theta}_o > E_o^{input^*}$. By contracting the given condition, assume that $\left(\overline{\alpha_o}, \bar{\lambda}, \bar{\mu}, \overline{s^-}\right)$ is not a Pareto solution. So, there must exist another feasible solution of *Model (6)* $\left(\hat{\alpha}_o, \hat{\lambda}, \hat{\mu}, \hat{s}^-\right)$ such that $\hat{\alpha}_o \le \bar{\alpha}_o$, $\hat{\alpha}_o \ne \bar{\alpha}_o$. According to the first half of the theorem, we can obtain the efficiency score of $DMU(\hat{\alpha}_o, \beta_o)$ is $E_o^{input^*}$. Since $\hat{\alpha}_o \le \bar{\alpha}_o$, $\hat{\alpha}_o \ne \bar{\alpha}_o$, we have $E_o^{input^*} = Eff\left(\hat{\alpha}_o, \beta_o\right) > Eff\left(\alpha_o, \beta_o\right) = \bar{\theta}_o$ which contradicts the given condition. Hence, the condition is wrong, which means is a Pareto solution.

Since *Model (6)* is an NLMOP, the following three-stage method is presented:

Stage 1. Solve *Model (5)* and suppose that $\left(E_o^{input^*}, \lambda^*, \mu^*, s^{-*}\right)$ is the optimal solution. Then, for $i = 1, 2, ..., m$ calculate $\theta_i^* = \dfrac{s_i^{-*}}{x_{io}}$.

Stage 2. Assume the output levels of DMU_o perturbs from y_o to $\beta_o = y_o + \Delta y_o$. Solve the following model with new output levels:

$$
\begin{aligned}
\overline{E}_o^{input} &= \min\ 1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}, \\
\text{s.t.}\quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, && i = 1, ..., m, \\
& \sum_{j=1}^{n} \mu_j y_{rj} \ge \beta_{ro}, && r = 1, ..., s, \\
& \sum_{j=1}^{n} \left(\lambda_j - \mu_j\right) z_{gj} \ge 0, && g = 1, ..., h, \\
& \lambda_j \ge 0, && j = 1, ..., n, \\
& \mu_j \ge 0, && j = 1, ..., n, \\
& s_i^- \ge 0, && i = 1, ..., m,
\end{aligned}
\tag{8}
$$

Assume that $\left(\overline{E}_o^{input^*}, \overline{\lambda}^*, \overline{\mu}^*, \overline{s^{-*}}\right)$ is an optimal solution of *Model (8)*.

Stage 3. Now, we are going to find the minimum levels of inputs in which the SBM efficiency scores of all DMUs remain unchanged by solving the following model:

$$
\begin{aligned}
\text{Min} \quad & \left(\alpha_{1o}, \alpha_{2o}, \ldots, \alpha_{mo}\right), \\
\text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \alpha_{io}, && i = 1,\ldots,m, \\
& \sum_{j=1}^{n} \mu_j y_{rj} \geq \beta_{ro}, && r = 1,\ldots,s, \\
& \sum_{j=1}^{n} \left(\lambda_j - \mu_j\right) z_{gj} \geq 0, && g = 1,\ldots,h, \\
& \theta_i^* = \sum_{i=1}^{m} \frac{\overline{s_i^{-*}}}{\alpha_i}, && i = 1,\ldots,m, \\
& \lambda_j \geq 0, && j = 1,\ldots,n, \\
& \mu_j \geq 0, && j = 1,\ldots,n, \\
& s_i^- \geq 0, && i = 1,\ldots,m, \\
& \alpha_{io} \geq 0, && i = 1,\ldots,m.
\end{aligned}
\tag{9}
$$

Theorem 2. Assume DMU_o perturbs its output level from y_o to $\beta_o = y_o + \Delta y_o$ $\left(\Delta y_o \geq 0,\ \Delta y_o \neq 0\right)$. If $\left(\lambda, \mu, \alpha\right)$ is a weak efficient solution of the MOLP *Model (9)*, then the input-oriented SBM efficiency score of all units stays unchanged after changing to the new input-output levels.

Proof: Since the input-oriented SBM model is a non-radial model, we can derive the results from *Theorem 1*.

4 | An empirical Illustration

In this section, we use the data set of Chen and Zhu [35] to illustrate the applicability of our proposed model and three-stage procedure. *Table 1* shows the data of 27 banks. The inputs, the intermediate products, and the outputs are introduced in *Table 1*. Fixed assets, IT budget, and the number of employees are three indicators that are considered as inputs (Second, third, and fourth column) to produce deposits as an intermediate product (Fifth column). Profit and fraction of loans recovered are the final outputs (Columns six and seven).

Table 1. The data set of Chen and Zhu [35].

Banks	Fixed Assets ($ billion) (x_1)	IT Budget ($ billion) (x_2)	# of Employees (Thousand) (x_3)	Deposits ($ billion) (z_1)	Profit ($ billion) (y_1)	Fraction of Loans Recovered (y_2)
1	0.713	0.15	13.3	14.478	0.232	0.986
2	1.071	0.17	16.9	19.502	0.34	0.986
3	1.224	0.235	24	20.952	0.363	0.986
4	0.363	0.211	15.6	13.902	0.211	0.982
5	0.409	0.133	18.485	15.206	0.237	0.984
6	5.846	0.497	56.42	81.186	1.103	0.955
7	0.918	0.06	56.42	81.186	1.103	0.986
8	1.235	0.071	12	11.441	0.199	0.985
9	18.12	1.5	89.51	124.072	1.858	0.972
10	1.821	0.12	19.8	17.425	0.274	0.983
11	1.915	0.12	19.8	17.425	0.274	0.983
12	0.874	0.05	13.1	14.342	0.177	0.985
13	6.918	0.37	12.5	32.491	0.648	0.945
14	4.432	0.44	41.9	47.653	0.639	0.979

Table 1. Continued.

Banks	Fixed Assets ($ Billion) (x_1)	IT Budget ($ Billion) (x_2)	# of Employees (Thousand) (x_3)	Deposits ($ Billion) (z_1)	Profit ($ Billion) (y_1)	Fraction of Loans Recovered (y_2)
15	4.504	0.431	41.1	52.63	0.741	0.981
16	1.241	0.11	14.4	17.493	0.243	0.988
17	0.45	0.053	7.6	9.512	0.067	0.98
18	5.892	0.345	15.5	42.469	1.002	0.948
19	0.973	0.128	12.6	18.987	0.243	0.985
20	0.444	0.055	5.9	7.546	0.153	0.987
21	0.508	0.057	5.7	7.595	0.123	0.987
22	0.37	0.098	14.1	16.906	0.233	0.981
23	0.395	0.104	14.6	17.264	0.263	0.983
24	2.68	0.206	19.6	36.43	0.601	0.982
25	0.781	0.067	10.5	11.581	0.12	0.987
26	0.872	0.1	12.1	22.207	0.248	0.972
27	1.757	0.0106	12.7	20.67	0.253	0.988

The results of the three-stage algorithm are depicted in *Table 2*. Let output levels of units be increased by 10 percent, and the vector $(w_1, w_2, w_3) = (1,1,1)$ is the input weights. Consider DMU25, with the lowest SBM efficiency score of 0.07750. By 10 percent perturbation in its output levels, the first input level will increase by 0.33312 units, and the second and third input levels will decrease by 0.05428 and 10.37880 units, respectively. So, the new input vector of DMU25 would be (1.11412, 0.01272, 0.12120). Now, consider DMU13, with the highest SBM efficiency score of 0.87100. When the output levels increase by 10 percent, all three input levels will decrease by 6.90819, 0.36225, and 12.37793 units, and the new input vector is (0.00981, 0.00775, 0.12207).

Now, we compare our inverse two-stage SBM model with the inverse two-stage cost efficiency model proposed by Shiri Daryani et al. [32]. The results are depicted in *Table 3*. Contemplate on DMU5 with SBM and CCR efficiency scores of 0.32127 and 0.507, respectively. We perturb its output levels by 10 percent and see that its first, second, and third input levels would decrease by 0.33339, 0.11985, and 18.35974 units, respectively, in order to preserve the SBM efficiency score at 0.32127. Through a 10 percent increase in DMU5's output levels, it is observed that its first and second input levels decrease by 0.004 and 0.074 units, respectively, and the third input increases by 0.023 levels, while its cost efficiency score remains unchanged.

Table 2. Results of inverse two-stage SBM model.

Banks	Inverse Two-Stage SBM								Fixed Asset Changes	IT Budget Changes	Number of Employees Changes
	SBM Efficiency	Input Slacks				New Input Levels					
	$E_o^{input^*}$	$s_1^{-^*}$	$s_2^{-^*}$	$s_3^{-^*}$	θ^*	α_1	α_2	α_3	Δx_1	Δx_2	Δx_3
1	0.32449	0.03103	0.09343	0.04511	0.89319	0.01241	0.00980	0.15439	-0.70059	-0.14020	-13.14561
2	0.36421	0.04694	0.10372	0.03739	0.87370	0.01373	0.01084	0.17084	-1.05727	-0.15916	-16.72916
3	0.28691	0.05456	0.14641	0.10647	0.89630	0.10477	0.02067	0.19694	-1.11923	-0.22416	-23.80306
4	0.32015	0.01247	0.13469	0.08013	0.91573	0.10321	0.01266	0.12058	-0.25979	-0.19834	-15.47942
5	0.32127	0.01408	0.08196	0.10082	0.86334	0.07561	0.01315	0.12526	-0.33339	-0.11985	-18.35974
6	0.33191	0.28315	0.30016	0.13912	0.89196	0.08009	0.04244	0.62442	-5.40591	-0.45456	-55.79558

Table 2. Continued.

Banks	Inverse Two-Stage SBM										
	SBM Efficiency	Input Slacks				New Input Levels			Fixed Asset Changes	IT Budget Changes	Number of Employees Changes
	$E_o^{input^*}$	$s_1^{-^*}$	$s_2^{-^*}$	$s_3^{-^*}$	θ^*	α_1	α_2	α_3	Δx_1	Δx_2	Δx_3
7	0.77928	0.01118	0.00883	0.13912	0.22072	0.03278	0.02588	0.40778	-0.88522	-0.03412	-56.01222
8	0.29547	0.06102	0.04170	0.04524	0.88937	0.02393	0.00907	0.12193	-1.21107	-0.06193	-11.87807
9	0.31548	0.93350	0.94749	0.17258	0.94049	0.92484	0.11000	1.04803	-17.19516	-1.39000	-88.46197
10	0.22252	0.24504	0.10564	0.24629	0.91508	0.31523	0.02934	0.27952	-1.50577	-0.09066	-19.52048
11	0.22138	0.25863	0.10564	0.24629	0.91799	0.32806	0.02934	0.27952	-1.58694	-0.09066	-19.52048
12	0.24869	0.11459	0.04072	0.16647	0.88077	0.15062	0.01859	0.17715	-0.72338	-0.03141	-12.92285
13	0.87100	0.19623	0.07058	0.00000	0.19476	0.00981	0.00775	0.12207	-6.90819	-0.36225	-12.37793
14	0.22005	0.70979	0.40651	0.47233	0.93634	0.89773	0.06977	0.66470	-3.53427	-0.37023	-41.23530
15	0.25544	0.71525	0.39217	0.38811	0.92638	0.68545	0.08106	0.77231	-3.81855	-0.34994	-40.32769
16	0.15515	0.19450	0.09727	0.53403	0.90998	0.60960	0.02590	0.24676	-0.63140	-0.08410	-14.15324
17	0.14225	0.06927	0.04739	0.29938	0.31720	0.01263	0.12034	0.12034	-0.43737	0.06734	-7.47966
18	0.48509	0.81588	0.25145	0.00000	0.79356	0.06170	0.04872	0.76766	-5.83030	-0.29628	-14.73234
19	0.15232	0.34690	0.11524	0.44179	0.94111	0.66280	0.02596	0.24730	-0.31020	-0.10204	-12.35270
20	0.20948	0.15549	0.04696	0.17434	0.91743	0.22682	0.01597	0.15213	-0.21718	-0.03903	-5.74787
21	0.16908	0.18136	0.05053	0.18888	0.94053	0.30106	0.01272	0.12120	-0.38680	-0.04428	-5.57880
22	0.16836	0.12256	0.08577	0.52660	0.88253	0.54614	0.02485	0.23677	0.17614	-0.07315	-13.86323
23	0.18120	0.12990	0.09019	0.52729	0.87549	0.52858	0.02818	0.26847	0.13358	-0.07582	-14.33153
24	0.23015	0.96003	0.17444	0.50272	0.94069	0.98842	0.06569	0.62584	-1.69158	-0.14031	-18.97416
25	0.07750	0.43651	0.06069	0.72731	0.97203	1.11412	0.01272	0.12120	0.33312	-0.05428	-10.37880
26	0.12628	0.47981	0.08698	0.74756	0.95050	1.07885	0.02652	0.25272	0.20685	-0.07348	-11.84728
27	0.21732	0.78268	0.00830	0.78268	0.78268	1.66986	0.02706	0.25785	-0.08714	0.01646	-12.44215

Table 3. Results of two-stage cost efficiency model [32].

Inverse Two-Stage Cost Efficiency							
CCR Efficiency	Cost Efficiency Score	New Input Levels			Fixed Asset Changes	IT Budget Changes	Number of Employees Changes
E_o^{input}	CE_o	α_1	α_2	α_3	Δx_1	Δx_2	Δx_3
0.659	0.463	0.014	0.011	0.173	-0.025	-0.089	0.024
0.749	0.543	0.018	0.014	0.222	-0.041	-0.099	0.034
0.573	0.419	0.025	0.020	0.310	-0.043	-0.137	0.042
0.535	0.353	0.016	0.012	0.194	-0.004	-0.128	0.019
0.507	0.404	0.018	0.015	0.229	-0.004	-0.074	0.023
0.689	0.514	0.063	0.050	0.784	-0.260	-0.282	0.154
0.779	0.779	0.056	0.044	0.693	0.005	0.004	0.063
0.586	0.474	0.013	0.011	0.167	-0.055	-0.037	0.033
0.647	0.400	0.113	0.089	1.407	-0.887	-0.911	0.407
0.418	0.355	0.048	0.038	0.059	-0.215	-0.082	0.122
0.415	0.351	0.048	0.038	0.600	-0.229	-0.082	0.127
0.432	0.385	0.030	0.024	0.372	-0.096	-0.026	0.059
0.900	0.541	0.053	0.042	0.654	-0.947	-0.328	0.356
0.438	0.336	0.106	0.084	1.324	-0.646	-0.356	0.324

Table 3. Continued.

Inverse Two-Stage Cost Efficiency							
CCR Efficiency	**Cost Efficiency Score**	**New Input Levels**			**Fixed Asset Changes**	**IT Budget Changes**	**Number of Employees Changes**
E_o^{input}	CE_o	α_1	α_2	α_3	Δx_1	Δx_2	Δx_3
0.507	0.390	0.107	0.084	1.328	-0.658	-0.347	0.328
0.257	0.237	0.069	0.054	0.859	-0.142	-0.056	0.124
0.220	0.206	0.035	0.028	0.442	-0.041	-0.025	0.054
0.795	0.581	0.092	0.073	1.145	-0.908	-0.272	0.354
0.280	0.238	0.063	0.050	0.789	-0.300	-0.078	0.146
0.382	0.324	0.029	0.023	0.365	-0.136	-0.032	0.064
0.315	0.259	0.029	0.023	0.356	-0.161	-0.034	0.065
0.259	0.243	0.066	0.052	0.818	-0.072	-0.046	0.099
0.282	0.264	0.068	0.054	0.849	-0.079	-0.050	0.104
0.422	0.327	0.099	0.078	1.237	-0.901	-0.128	0.237
0.118	0.098	0.074	0.059	0.926	-0.370	-0.008	0.099
0.208	0.175	0.087	0.069	1.087	-0.409	-0.031	0.135
0.217	0.151	0.085	0.067	1.060	-0.915	0.057	0.060

5 | Conclusion

Since the conventional InvDEA models estimate inputs/outputs of DMUs without considering slacks, in this paper, an inverse two-stage SBM model was proposed for input estimation of units with a two-stage network structure in order to keep the non-radial efficiency score unchanged. The existence of slacks gives more accurate information about inputs/outputs of under-evaluation DMUs and estimates inputs/outputs of DMUs more accurate and real. Since the proposed model was an NLMOP, a three-stage algorithm was proposed for the input estimation procedure. In the model, the DM's preferences can be considered in input weights in the input estimation procedure. Finally, the proposed method was applied to an empirical example in a two-stage network in the banking industry with three inputs, one intermediate product, and two outputs.

This study can be extended to several directions in the future. For example, the current method is based on the inverse non-radial SBM model, and future study prefers to develop it in the other non-radial models. Another future research line may be working with other DEA networks and supply chain structures. In addition, it can be extended to the two-stage networks with imprecise or negative data.

Author Contribution

The author was involved in conceptualization, model development, software implementation, validation of the model and its execution, sensitivity analysis, and paper writing.

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All data are included in this research article.

Conflicts of Interest

The author declares no conflict of interest.

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