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A New Inverse DEA Method for Input Estimation and Efficiency Improvement in Network Systems by a Two-Stage Structure

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Abstract

Inverse Data Envelopment Analysis (InvDEA) is a helpful tool for estimating the inputs/outputs of Decision-Making Units (DMUs) in order to achieve an unchanged efficiency score. This paper first provides an InvDEA method for input estimation in the two-stage network systems when the efficiency score of all units is fixed. Then, an InvDEA method is developed in order to improve the efficiency of all DMUs. Finally, a case study is presented to show the capabilities of the proposed methods.

Keywords: Data envelopment analysis, Inverse data envelopment analysis, Network data envelopment analysis, Two-stage network, Input estimation, Efficiency improvement.

1 | Introduction

Data Envelopment Analysis (DEA) is a nonparametric method for assessing the efficiency score of Decision-Making Units (DMUs) with multiple inputs and outputs. However, the Inverse Data Envelopment Analysis (InvDEA) concept proposed by Wie et al. [1] is a method to answer these questions: If among a group of comparable DMUs, we increase the input level of a certain DMU, how much more output would be produced so that the efficiency score of all DMUs stays unchanged? Or if the output level of a certain DMU perturbs the current level, how many more inputs are required in order to increase the unchanged efficiency score of all DMUs? There are many studies to answer these questions.

Yan et al. [2] discussed the InvDEA problem with preference cone constraints. Moreover, Jahanshahloo et al. [3] used the InvDEA model and proposed a model in order to improve the current efficiency level. In addition, Letrtworasirikul et al. [4] developed the inverse BCC model, which can preserve the relative

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efficiency values of all DMUs. Furthermore, Eyni et al. [5] applied InvDEA and cone constraints to the sensitivity analysis of DMUs with undesirable inputs and outputs. Moreover, Amin et al. [6] combined InvDEA and goal programming methods for target setting in mergers.

The concept of network DEA is used to evaluate the efficiency score of systems with a network structure. A special case of networks is the basic two-stage network, in which exogenous inputs enter the system as the inputs of the first stage to produce intermediate products as inputs of the second stage to produce final outputs. Seiford and Zhu [7] proposed a two-stage network DEA concept. They used the standard DEA model to evaluate the efficiency score of each stage and the whole system from an independent point of view. They did not consider the relationship between divisions. Thus, Kao and Hwang [8] proposed a model that evaluates the efficiency score of the whole system and each division by considering their relationship.

In this paper, the InvDEA concept is used to estimate the input level of DMUs with a two-stage network structure in order to increase the unchanged efficiency score of each stage and the whole system. Furthermore, an InvDEA method would be proposed for input estimation with respect to the efficiency improvement of units.

The rest of this paper is as follows: in Section 2, we point out some basic concepts of DEA, InvDEA, and a basic two-stage network. An InvDEA method for input estimation in two-stage network systems will be proposed in Section 3. In Section 4, an invDEA method for input estimation of units will be proposed in order to improve the efficiency of units. An examination of the proposed method in non-life insurance companies in Taiwan will be presented in Section 5.

2 | Preliminary

2.1 | Data Envelopment Analysis

DEA is a nonparametric technique to assess the efficiency score of DMUs with multiple inputs and outputs proposed by Charnes et al. [9]. Assume there are n DMUs $(DMU_j, j = 1, \dots, n)$ which consume m inputs $(x_{ij}, i = 1, \dots, m)$ to produce s outputs $(y_{rj}, r = 1, \dots, s)$. The following model measures the input-oriented efficiency score of the evaluation unit (x_o, y_o) which is called DMU_o:

$$\begin{aligned}
&\theta_o^* = \min \quad \theta, \\
&\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io}, \ i = 1, \dots, m, \\
&\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \ r = 1, \dots, s, \\
&\lambda_j \ge 0, \ j = 1, \dots, n.
\end{aligned} \tag{1}$$

Definition 1. If in the optimal solution of *Model (1)*, $\theta_o^* = 1$, then DMU_o called CCR efficient, otherwise DMU_o is inefficient.

2.2 | Inverse Data Envelopment Analysis

The concept of InvDEA proposed by Wei et al. [1] to answer this question: If among a group of comparable DMUs, the outputs of a particular DMU are revised, how many more inputs are required in order to maintain the efficiency score of all DMUs? To answer this question, assume the output level of DMU_o perturbs from y_o to $\beta_o = y_o + \Delta y_o$. The InvDEA model, which estimates the input level $\alpha_i = x_i + \Delta x_i (i = 1, \dots, m)$ of DMUs and guarantee the unchanged efficiency score of all DMUs is:

$$\alpha_i^* = \min\{\alpha_1, \alpha_2, \dots \alpha_m\},$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o^* \alpha_i, \ i = 1, \dots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \beta_{ro}, \ r = 1, \dots, s, \tag{2}$$

$$\lambda_j \ge 0, \ j = 1, \dots, n.$$

Note that θ_o^* calculated by *Model (1)*.

Since *Model (2)* is an MOLP, assume $w_i (i = 1, \dots, m)$ is the weight (value, price) of unit i. Now, it can be easily solved by following a single objective model:

$$\alpha_i^* = \min \sum_{i=1}^{m} w_i \alpha_i,$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o^* \alpha_i, \ i = 1, \dots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \beta_{ro}, \ r = 1, \dots, s, \tag{3}$$

$$\lambda_j \ge 0, \ j = 1, \dots, n.$$

Now consider this question: If among a group of DMUs, we increase specific inputs to a particular unit and assume that its current efficiency level with respect to other units is improved, say, t-percent of θ_o^*, how much output could the unit produce? To answer this question, put $\theta_o^* + \left(\frac{t}{100}\right)\theta_o^*$ instead of θ_o^* in the *Model (3)*. Then, the *Model (3)* is converted to the following form:

$$\alpha_i^* = \min\{\alpha_1, \alpha_2, \dots \alpha_m\},$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \left(1 + \frac{t}{100}\right) \theta_o^* \alpha_i, \ i = 1, \dots, m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge \beta_{ro}, \quad r = 1, \dots, s, \tag{4}$$

$$\lambda_j \ge 0, \quad j = 1, \dots, n.$$

3|An Inverse Data Envelopment Analysis Method for Inputs Estimation of a Two-Stage Process When Outputs of the Whole System Are Revised

In this section, according to the concepts of InvDEA and network DEA, we propose a method for input estimation in the basic two-stage network systems in order to unchanged efficiency scores of Stage 1, Stage 2, and the whole system. We want to do this work with an independent perspective on two-stage network systems. The main issue is to answer this question: if the final output level of DMU_o with the two-stage structure, the increase from y_o to $\beta_o = y_o + \Delta y_o$, how many more inputs $\alpha_i = x_i + \Delta x_i (i = 1, \ldots, m)$ are required in order for the efficiency scores of Stage 1, Stage 2, and the whole system to stay unchanged. To overcome this issue, the following steps are recommended:

Step 1. Calculate the efficiency score of Stage 2 (θ_2^*) by solving the following model (Note that the intermediate products of the whole system are the inputs of Stage 2, and the final outputs of the entire system are the outputs of Stage 2):

$$\theta_2^* = \min \theta_2,$$
$$\text{s.t.} \sum_{j=1}^{n} \lambda_j z_{gj} \leq \theta_2 z_{go}, \quad g = 1, \ldots, h,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \ldots, s, \tag{5}$$
$$\lambda_j \geq 0, \quad j = 1, \ldots, n.$$

Assume the output level of Stage 2 perturbs from y_{ro} to $y_{ro} + \Delta y_{ro} = \beta_{ro} (r = 1, \ldots, s)$. Now we estimate the input level of this stage in order to calculate the efficiency score, θ_2^* stays unchanged by solving the ongoing MOLP model:

$$\gamma_g^* = \min\{\gamma_1, \gamma_2, \ldots, \gamma_h\},$$
$$\text{s.t.} \sum_{j=1}^{n} \lambda_j z_{gj} \leq \theta_2^* \gamma_g, \quad g = 1, \ldots, h,$$
$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \Delta y_{ro} = \beta_{ro}, \quad r = 1, \ldots, s, \tag{6}$$
$$\lambda_j \geq 0, \quad j = 1, \ldots, n.$$

Since the above model is an MOLP, as mentioned before, it can be inverted to a single-objective problem.

Step 2. First, calculate the efficiency score of Stage 1 (θ_1^*) by solving the following model (Note that the intermediate products of the whole system are the outputs of Stage 1 and the inputs of the entire system are the inputs of Stage 1):

$$\theta_1^* = \min \theta_1,$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_1 x_{io}, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j z_{gj} \geq z_{go}, \quad g = 1, \ldots, h, \tag{7}$$

$$\lambda_j \geq 0, \quad j = 1, \ldots, n.$$

Note that the new output level of Stage 1, $\gamma_{go} (g = 1, \ldots, h)$ are driven in *Step 1*. Then we estimate the input level of Stage 1 (Simultaneously, the inputs of the whole system) in order to unchanged the efficiency score of Stage 1 (θ_1^*).

$$\alpha_i^* = \min\{\alpha_1, \alpha_2, \ldots, \alpha_m\},$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_1^* \alpha_i, \quad i = 1, \ldots, m,$$

$$\sum_{j=1}^{n} \lambda_j z_{gj} \geq z_{go} + \Delta z_{go} = \gamma_{go}, = 1, \ldots, h, \tag{8}$$

$$\lambda_j \geq 0, \quad j = 1, \ldots, n.$$

4 | Efficiency Improvement

In this section, we deal with this question: if among a group of DMUs with a two-stage structure, we increase the output level of the whole system and assume that the current efficiency level of stage 1 and stage 2 increases by t_1 percent of θ_1^* and t_2 percent of θ_2^*, respectively, how many more inputs does the whole system require? To answer the question, follow the following steps:

Step 1. Find the efficiency scores of Stage 1 and Stage 2 by *Model (7)* and *Model* (5), respectively.

Step 2. Put $\theta_2^* + \left(\frac{t_2}{100}\right)\theta_2^* = \left(1 + \frac{t_2}{100}\right)\theta_2^*$ instead of θ_2^* in *Model (6),* as follows, and find the minimum level of inputs of Stage 2 (Intermediate products of the whole system) $\gamma_g (g = 1, \ldots, h)$ by solving the following model:

$$\gamma_g^{\text{improved}} = \min\{\gamma_1, \gamma_2, \ldots, \gamma_h\},$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \lambda_j z_{gj} \leq \left(1 + \frac{t_2}{100}\right)\theta_2^* \gamma_g, \quad g = 1, \ldots, h,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \Delta y_{ro} = \beta_{ro}, \quad r = 1, \ldots, s, \tag{9}$$

$$\lambda_j \geq 0, \quad j = 1, \ldots, n.$$

Step 3. Put $\theta_1^* + \left(\frac{t_1}{100}\right)\theta_1^* = \left(1+\frac{t_1}{100}\right)\theta_1^*$ instead of θ_2^* in *Model (7),* as follows, and find the minimum level of inputs of Stage 1 (Inputs of the whole system) $\alpha_i(i = 1, \ldots, m)$ by solving the following model:

$$\begin{aligned}
&\alpha_i^{\text{improved}} = \min\{\alpha_1, \alpha_2, \ldots, \alpha_m\},\\
&\text{s.t.}\ \ \sum_{j=1}^{n} \lambda_j x_{ij} \le \left(1+\frac{t_1}{100}\right)\theta_1^* \alpha_i, \quad i = 1, \ldots, m,\\
&\sum_{j=1}^{n} \lambda_j z_{gj} \ge z_{go} + \Delta z_{go} = \gamma_{go}, \qquad g = 1, \ldots, h,\\
&\lambda_j \ge 0, \qquad j = 1, \ldots, n.
\end{aligned} \tag{10}$$

5 | Case Study

In this section, we examine our developed methods on an empirical illustration of 24 non-life insurance companies in Taiwan used by Kao and Hwang [8], which are depicted in *Table 1*. Operation expenses (x_1) and insurance expenses (x_2) are the inputs of Stage 1 and the whole system. Underwriting profit (z_1) and investment profit (z_2) are treated as intermediate products of the whole system (Outputs of Stage 1 and inputs of Stage 2). Direct written premiums (y_1) and reinsurance premiums (y_2) are outputs of Stage 2 and the whole system. The data set is shown in *Table 1*.

Table 1. The data set of Kao and Hwang [8].

DMU	Operation Expenses (x_1)	Insurance Expenses (x_2)	Under Writing Profit (z_1)	Investment Profit (z_2)	Direct Written Premium (y_1)	Reinsurance Premiums (y_2)
Taiwan fire	1,178,744	673,512	7,451,757	856,735	984,143	681,687
Chung Kuo	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
Tai Ping	1,177,494	592,790	4,776,548	560,244	293,613	658,428
China Mariners	601,320	594,259	3,174,851	371,863	248,709	177,331
Fubon	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
Zurich	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
Taian	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
Ming Tai	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
Central	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
The First	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
Kuo Hua	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
Union	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
Shingkong	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
South China	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
Cathay Century	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
Allianz president	1,211,716	415,071	5,606,013	402,881	854,054	197,947

Table 1. Continued.

DMU	Operation Expenses (x_1)	Insurance Expenses (x_2)	Under Writing Profit (z_1)	Investment Profit (z_2)	Direct Written Premium (y_1)	Reinsurance Premiums (y_2)
Newa	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
AIU	757,515	547,997	3,631,484	995,620	692,731	163,927
North America	159,422	182,338	1,141,950	483,291	519,121	46,857
Federal	145,442	53,518	316,829	131,920	355,624	26,537
Royal & Sunalliance	84,171	26,224	225,888	40,542	51,950	6491
Asia	15,993	10,502	52,063	14,574	82,141	4181
AXA	54,693	28,408	245,910	49,864	0.1	18,980
Mitsui Sumitomo	163,297	235,094	476,419	644,816	142,370	16,976

The results are depicted in *Table 2*. The second and third column of *Table 2* shows the efficiency scores of Stage 1 and Stage 2, respectively. The efficiency score of the whole system is depicted in the fourth column of *Table 2*. DMUs 1, 2, 9, and 12 are CCR efficient in stage 1. DMUs 3 and 5 are CCR efficient in stage 2.

Table 2. CCR efficiencies of stage 1, stage 2, and the whole system.

DMU	θ_1^*	θ_2^*	θ
Taiwan fire	1.00	0.71	0.71
Chung Kuo	1.00	0.60	0.60
Tai Ping	0.70	1.00	0.70
China Mariners	0.72	0.43	0.31
Fubon	0.84	1.00	0.84
Zurich	0.96	0.41	0.39
Taian	0.75	0.54	0.40
Ming Tai	0.73	0.51	0.37
Central	1.00	0.29	0.29
The First	0.86	0.67	0.58
Kuo Hua	0.75	0.33	0.25
Union	1.00	0.76	0.76
Shingkong	0.81	0.54	0.44
South China	0.72	0.52	0.38
Cathay Century	1.00	0.70	0.70
Allianz President	0.91	0.38	0.35
Newa	0.72	1.00	0.72
AIU	0.92	0.34	0.32
North America	0.97	1.00	091
Federal	1.00	0.80	0.80
Royal and Sunalliance	0.77	0.28	0.22
Asia	0.68	1.00	0.68
AXA	0.95	0.54	0.51
Mitsui Sumitomo	1.00	0.18	0.18

Assume the output level of the whole system (Outputs of Stage 2) increased by 10%. Consider DMU1, an efficiency score of 1.00 in Stage 1 and 0.71 in Stage 2. If we increase the outputs of Stage 2 and the efficiency of the whole system by 10%, we see that the new intermediate products vector of DMU1 is (7909380.29, 1038320.22) and the requirement inputs vector is (1087020.49, 914130.28).

Table 3. New level of intermediate products and inputs, intermediate products changes, and inputs changes.

DMU	New Level of Underwriting Profit (γ_1)	Under Writing Profit Changes (Δz_1)	New Level of Investment Profit (γ_2)	Investment Profit Changes (Δz_2)	New Level of Operation Expenses (α_1)	Operation Expenses Changes (Δx_1)	New Level of Insurance Expenses (α_2)	Insurance Expenses Changes (Δx_2)
Taiwan fire	7909380.29	457623.29	1038320.22	181585.22	1087020.49	-91723.51	914130.28	240618.28
Chung Kuo	1.14538E+7	1433547.82	1507951.37	-304942.63	1574185.65	192363.65	1325472.29	-27282.71
Tai Ping	5254202.80	477654.80	616268.40	56024.40	1023485.21	-154008.79	820730.99	227940.99
China Mariners	3390092.58	215241.58	443152.13	71289.13	642686.99	41366.99	539468.75	-54790.25
Fubon	3.31401E+7	-4.2528E+6	4594882.32	2841088.32	5440963.67	-1.2581E+6	4687428.32	1155814.32
Zurich	9286916.91	-460991.09	1526456.70	574130.70	1327440.36	-1.3003E+6	1238557.51	570194.51
Taian	7645643.42	-3.0398E+6	1337670.79	694258.79	1401142.94	-541690.06	1348506.41	-94593.59
Ming Tai	1.18294E+7	-5.4378E+6	2209280.14	1074680.14	2261677.07	-1.5273E+6	2226901.93	353371.93
Central	8162821.90	-3.3103E+6	1325422.09	779085.09	1124248.16	-443497.84	1042746.98	92314.98
The First	7213824.42	-996564.58	1095965.87	591437.87	1152212.73	-151036.27	1035225.46	-263244.54
Kuo Hua	3171552.78	-4.0508E+6	887813.04	244635.04	655043.35	-1.3074E+6	697540.94	25126.94
Union	1.00110E+7	576549.67	1352712.59	234223.59	1376212.58	-1.2166E+6	1172087.50	521135.50
Shingkong	5197967.61	-8.7235E+6	1358952.63	547609.63	973679.68	-1.6363E+6	1022672.73	-346129.27
South China	5841283.14	-1.5551E+6	965891.25	500382.25	1110280.90	-285721.10	1038989.55	50101.55
Cathay Century	7602978.38	-2.8193E+6	1403373.95	653480.95	1053333.84	-1.1316E+6	1034923.28	383860.28
Allianz President	4698275.80	-907737.20	781999.32	379118.32	713057.21	-498658.79	669425.27	254354.27
Newa	3855390.10	-3.8401E+6	797799.58	455310.58	759272.65	-694524.35	761970.75	-323048.25
AIU	4330674.13	699190.13	716536.58	-279083.42	645110.76	-112404.24	603867.41	55870.41
North America	522279.96	380329.96	119066.20	-364224.80	78974.26	-80447.74	80737.69	-101600.31
Federal	392260.31	75431.31	95817.13	-36102.87	58435.13	-87006.87	60552.35	7034.35
Royal and Sunalliance	237582.69	11694.69	48229.88	7687.88	43544.88	-40626.12	43542.29	17318.29
Asia	57269.30	5206.30	16031.40	1457.40	13027.62	-2965.38	13872.82	3370.82
AXA	280311.02	34401.02	32877.84	-16986.16	40616.51	-14076.49	32570.30	4162.30
Mitsui Sumitomo	974994.15	498575.15	201194.60	-443621.40	138786.69	-24510.31	139206.62	-95887.38

Now, the decision maker wants to increase the efficiency scores of Stage 1 and Stage 2 by 20% and 30%, respectively. The new level of intermediate products and inputs is depicted in *Table 4*.

Table 4. New level of intermediate products and inputs, intermediate products changes, and inputs changes after improvement of the efficiency scores of Stage 1 and Stage 2 by 20% and 30%, respectively.

DMU	$\gamma_1^{improved}$	$\Delta z_1^{improved}$	$\gamma_2^{improved}$	$\Delta z_2^{improved}$	$\alpha_1^{improved}$	$\Delta x_1^{improved}$	$\alpha_2^{improved}$	$\Delta x_2^{improved}$
Taiwan fire	6084138.68	-1.3676E+6	798707.86	-58027.14	1294072.01	115328.01	1088250.33	414738.33
Chung Kuo	8810611.40	-1.2096E+6	1159962.59	-652931.41	1009093.37	-372728.63	849661.73	-503093.27
Tai Ping	4041694.46	-734853.54	474052.62	-86191.38	656080.26	-521413.74	526109.61	-66680.39
China Mariners	2607763.52	-567087.48	340886.25	-30976.75	411978.84	-189341.16	345813.30	-248445.70
Fubon	2.54924E+7	-1.1900E+7	3534524.86	1780730.86	3487797.22	-3.2113E+6	3004761.75	-526852.25
Zurich	7143782.24	-2.6041E+6	1174197.46	221871.46	850923.31	-1.7768E+6	793947.12	125584.12

Table 4. Continued.

DMU	$\gamma_1^{improved}$	$\Delta z_1^{improved}$	$\gamma_2^{improved}$	$\Delta z_2^{improved}$	$\alpha_1^{improved}$	$\Delta x_1^{improved}$	$\alpha_2^{improved}$	$\Delta x_2^{improved}$
Taian	5881264.17	-4.8042E+6	1028977.53	385565.53	898168.55	-1.0447E+6	864427.19	-578672.81
Ming Tai	9099574.44	-8.1677E+6	1699446.26	564846.26	1449792.99	-2.3392E+6	1427501.24	-446028.76
Central	6279093.77	-5.1941E+6	1019555.46	473218.46	720671.90	-847074.10	668427.55	-282004.45
The First	5549095.71	-2.6613E+6	843050.67	338522.67	738597.91	-564651.09	663606.06	-634863.94
Kuo Hua	2439655.99	-4.7827E+6	682933.11	39755.11	419899.58	-1.5425E+6	447141.63	-225272.37
Union	7700735.13	-1.7337E+6	1040548.15	-77940.85	882187.55	-1.7106E+6	751338.14	100386.14
Shingkong	3998436.62	-9.9230E+6	1045348.17	234005.17	624153.64	-1.9858E+6	655559.44	-713242.56
South China	4493294.72	-2.9031E+6	742993.27	277484.27	711718.53	-684283.47	666018.94	-322869.06
Cathay Century	5848444.91	-4.5739E+6	1079518.43	329625.43	675214.00	-1.5097E+6	663412.36	12349.36
Allianz President	3614058.31	-1.9920E+6	601537.94	198656.94	457087.96	-754628.04	429118.76	14047.76
Newa	2965684.69	-4.7298E+6	613691.98	271202.98	486713.24	-967083.76	488442.79	-596576.21
AIU	3331287.79	-300196.21	551181.98	-444438.02	413532.54	-343982.46	387094.49	-160902.51
North America	401753.82	259803.82	91589.38	-391701.62	50624.53	-108797.47	51754.93	-130583.07
Federal	301738.70	-15090.30	73705.48	-58214.52	37458.42	-107983.58	38815.61	-14702.39
Royal and Sunalliance	182755.92	-43132.08	37099.90	-3442.10	27913.39	-56257.61	27911.73	1687.73
Asia	44053.31	-8009.69	12331.85	-2242.15	8351.04	-7641.96	8892.83	-1609.17
AXA	215623.86	-30286.14	25290.64	-24573.36	26036.22	-28656.78	20878.40	-7529.60
Mitsui Sumitomo	749995.50	273576.50	154765.08	-490050.92	88965.83	-74331.17	89235.01	-145858.99

Conflict of Interest

The authors declare that there is no conflict of interest regarding the publication of this article.

Data Availability

All data generated or analyzed during this study are included in this published article. No additional data are available.

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