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Determining Ranking Ranges Using Goal Programming in DEA

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Abstract

Data Envelopment Analysis (DEA) is a powerful non-parametric method used to evaluate the relative efficiency of Decision-Making Units (DMUs). The cross-efficiency method has been introduced as an extension of DEA, enabling each unit to be evaluated not only by its own optimal weights but also by the weights of its peers. By integrating DEA and the cross-efficiency method, a more reliable ranking of DMUs can be achieved, enhancing the discriminative power of the evaluation and supporting better decision-making. Alcaraz et al. [1] proposed a method to determine the ranking range of DMUs within the cross-efficiency evaluation. Their proposed models were non-linear. In this article, we use the goal-programming method and convert the nonlinear models into LPs to explore the best and worst ranks for each DMU. Our proposed method and the presented linear models are easier to solve and require less time and computation for systems.

Keywords: Data envelopment analysis, Cross-efficiency, Ranking ranges, Goal-programming, Best and worst rank.

1 | Introduction

In recent years, there has been a wide range of Data Envelopment Analysis (DEA) applications for evaluating the performance of entities engaged in diverse activities, contexts, and countries. DEA is a non-parametric method based on Linear Programming (LP) to determine efficient and inefficient Decision-Making Units (DMUs) and compare their efficiencies. Performance evaluation is typically expressed as a ratio, such as output/input. This is commonly referred to as an efficiency measure, which is less than or equal to 1. Measuring total performance, especially when there are multiple inputs and outputs, presents challenges such as selecting appropriate weights for the inputs and outputs to calculate the output/input ratio for efficiency measurement [2]. These weights are chosen so that each is evaluated under the most favorable conditions.

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The DEA model was first proposed by Charnes et al. [2] to obtain efficiency scores for DMUs (the CCR model) and to identify efficient and inefficient units. Subsequently, Banker et al.[3] introduced the BCC model, and others have since extended it. Another important topic for managers is ranking DMUs by performance. Many studies have been conducted in this area; for example, Andersen and Petersen [4] proposed a ranking method. They removed the DMU under evaluation from the Production Possibility Set (PPS) and applied the model to the remaining DMUs. However, this approach is infeasible and unstable for specific data sets.

Mehrabian et al. [5] later suggested an LP model, known as the MAJ model, for ranking efficient units in the presence of zero data. However, the MAJ model fails in some cases, as demonstrated by a theorem in [5]. In the MAJ model, unlike the AP model, movement towards the frontier was performed along the input axis with input orientation and equal steps. This approach removed the instability problem but caused infeasibility in some data sets. To address this weakness, Saati et al. [6] modified the MAJ model and proved that this version is always feasible and simultaneously both input- and output-oriented. Jahanshahloo et al. [7] proposed a ranking method using super-efficiency and the L1-norm. Jahanshahloo et al. [7] proposed a method for ranking efficient units. Khodabakhshi and Aryavash [8] proposed a ranking method in which the maximum and minimum efficiency scores for each unit are first computed under the assumption that all units have equal efficiency scores of 1. Then, by combining the maximum and minimum efficiency scores, the ranking is determined. Jahanshahloo et al. [9] proposed two new models to rank efficient units based on the L1-norm and input-output weights.

Ziari and Raissi [10] introduced a new method to rank extremely efficient DMUs in DEA by minimizing the distance between the virtual DMU and the DMU under evaluation. Ziari [11] introduced another method that transformed the non-LP model proposed by Jahanshahloo et al. [12] into an LP model to rank DMUs.Banhidi and Dobos [13] applied a common-weight DEA model to rank Central and Eastern European countries based on digital readiness indicators. Chen [14] proposes an extended version of super-efficiency DEA to achieve a complete and stable ranking among DMUs.

Cross-efficiency evaluation is an advanced extension of DEA designed to enhance the discrimination and ranking of DMUs. Unlike traditional DEA models, which allow each DMU to choose its own optimal weights, the cross-efficiency method introduces a peer-evaluation process in which each unit is also assessed using the weights of others. Therefore, the relative efficiency for each DMU is obtained by averaging all attained efficiencies, allowing DMUs to be ranked. This approach provides more reliable and fair efficiency scores by considering both self- and peer-assessments. Cross-efficiency evaluation has been used in various contexts, and some applications of this methodology can be found in research by Sexton et al. [15], who proposed a cross-efficiency method in which, using DEA, the optimal weights for the multiplier model were computed. Other studies concerning ranking methods in DEA include those by [12], [16-18].

Liu et al. [19] extended cross-efficiency evaluation to a two-stage DEA framework incorporating dual fairness constraints, providing a comprehensive and equitable efficiency analysis for multi-process decision-making systems. Kumar and Al-Hassan [20] applied cross-efficiency DEA and bootstrapped regression to measure how mobile e-learning influences school management efficiency. Orkcu [21] developed goal-programming models for use in the second stage of cross-efficiency evaluation in DEA to reduce weight multiplicity and improve discrimination power. Davtalab [22] introduced a novel secondary objective for cross-efficiency evaluation that maximizes the number of DMUs that reach their target efficiency.

The main problem in evaluating cross-efficiency is the possibility of multiple optimal DEA weightings, which may yield different rankings of DMUs. To address this issue, the use of secondary goals to determine weights was suggested. Sexton et al. [15] and Doyle and Green [23] proposed aggressive and benevolent formulations. These are examples of models that use an additional criterion to select weights. Extensions of these models are found in studies conducted by [24] and [17], [18]. Alcaraz et al. [1] proposed a method to determine the ranking range of DMUs within the cross-efficiency evaluation framework. They calculated the maximum and minimum cross-efficiency scores for each unit, and their results are more stable than those of traditional DEA models. In this article, the weights are determined through LP, as in traditional DEA models. However, instead of using a single set of optimal weights, the authors compute the maximum and minimum cross-efficiency scores for each unit by considering all feasible weight combinations. This approach treats weights as ranges rather than fixed values; therefore, a single rank for each unit cannot be achieved. The rank of each DMU is determined by the best and worst rankings that the unit could attain. One drawback of their proposed models is their non-linearity. In this article, we utilize the goal-programming method and convert the models proposed by Alcaraz et al. [1] into linear models. The advantage of our models is their ease of solution due to linearity, and the worst and best rankings for each unit are obtained using the same models.

The rest of the paper is organized as follows. Section 2 explains not only the computation of the cross-efficiency scores for each DMU but also illustrates the method introduced by Alcaraz et al. [1] for obtaining ranking ranges. Section 3 demonstrates the goal-programming procedure for computing ranking ranges. Section 4 presents the numerical example, and Section 5 provides the conclusion.

2 | Preliminaries

2.1 | Data Envelopment Analysis Models

Consider a set of peers observed DMUs (DMU_j, $j=1,...,n$) such that each DMU_j produces multiple non-negative outputs $y_{rj}\,(r=1,...,s)$ utilizing multiple non-negative inputs $x_{ij}\,(i=1,...,m)$. It is supposed that $x_j=(x_{1j},...,x_{mj})^T \neq 0_m$ and $\mathbf{y}_j=(y_{1j},...,y_{sj})^T \neq \mathbf{0}_s$ for each j. Moreover, assume that $D_j=(\mathbf{x}_j,\mathbf{y}_j)^T$ expresses the input and output vectors of each DMU_j, $j \in J=\{1,...,n\}$. It is assumed that there is no duplicate DMU. The PPS is defined as the set of all possible input-output vectors as follows:

$$PPS = \{(x, y): x \text{ can produce by } y\}.$$

The following problem computes the efficiency of DMUP:

$$\begin{aligned} \text{Max} \quad & \theta_p = \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}}, \\ \text{s. t.} \quad & \frac{\sum_{j=1}^{n} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \qquad j=1,...,n, \\ & v_i, u_r \geq 0, \qquad i=1,...,m, r=1,...,s, \end{aligned} \tag{1}$$

where $v_p=(v_{1p},...,v_{mp})$, $u_p=(u_{1p},...,u_{sp})$ are input and output weights, respectively. Using the method of converting linear fractional programming into LP suggested by [7], the above-mentioned *Model (1)* transforms into the following LP *Model (2),* which is named CCR proposed by [2]:

$$\begin{aligned} & \max \theta = \sum_{r=1}^{s} u_r y_{rp}, \\ \text{s. t.} \quad & \sum_{i=1}^{m} v_i x_{ip} = 1, \\ & -\sum_{i=1}^{m} v_i x_{ip} - \sum_{r=1}^{s} u_r y_{rj} \leq 0, \qquad j=1,...,n, \\ & u_r \geq 0, v_i \geq 0, \qquad r=1,...,s, i=1,...,m. \end{aligned} \tag{2}$$

Definition 1. $DMU_o=(x_o, y_o)$ is a CCR-efficient unit if and only if the optimal objective function of *Model (2)* is equal to one; otherwise, it is inefficient.

2.2 | Cross-Efficiency

Based on the optimal solutions of *Model (2)*, the cross-efficiency evaluation of each DMU can be obtained. If $(v_p^*, u_p^*) = (v_{1p}^*,...,v_{mp}^*, u_{1p}^*,...,u_{sp}^*)$ is an optimal solution of *Model (2)* for a given DMU_p, then the cross-efficiency of DMU_j , j=1,…,n obtained with the weights of DMU_p is the following:

$$\theta_{pj} = \frac{\sum_{r=1}^{s} u_{rp}^* y_{rj}}{\sum_{i=1}^{m} v_{ip}^* x_{ij}}, \qquad j = 1,...,n. \tag{3}$$

Then, the cross-efficiency score of DMU_j , j=1,…,n is usually defined as the average of the weights of all DMUs. It means the cross-efficiency score of DMU_j is determined as:

$$\bar{\theta}_j = \frac{\sum_{p=1}^{n} \theta_{pj}}{n}, \quad j = 1,...,n. \tag{4}$$

It is noticed that DEA models may have multiple optimal solutions. This non-uniqueness of input/output optimal weights would undermine the cross-efficiency concept, as it creates ambiguity in the use of weights for the computation of final results. In addition, this same problem (multiple optimal weights) sometimes causes a DMU not to have a unit rating.

2.3 | Cross-Efficiency and Ranking

In classical DEA and cross-efficiency evaluation, each DMU selects input and output weights to maximize its own efficiency. Sometimes, multiple optimal weights can exist, leading to unstable rankings depending on the chosen weight. To cope with this problem, Alcaraz et al. [1] proposed a method that, instead of assigning a single rank to each DMU, calculates a range of possible ranks showing the best and worst positions a DMU can occupy under all possible optimal weight selections. They defined the best and worst ranks for each unit as follows: the Best (Worst) rank is the least significant (most significant) achievable across all feasible optimal weight sets. To achieve the best and worst ranks, they suggested separate integer models.

The best ranking of a DMU_o (DMU under evaluation) is obtained with *Eq. (5)*, which is mentioned below:

$$r_o^b = n - WE_o^*, \tag{5}$$

where WE_o^* is the optimal solution of the *Model (6)* expressed as:

$$\max \quad WE_o = \sum_{j \neq o} I_j, \tag{6}$$

$$\text{s. t.} \qquad \frac{u_p^* y_p}{v_p^* x_p} = \theta_p^*, \qquad p = 1,...,n, \tag{6.1}$$

$$\frac{u_p^* y_j}{v_p^* x_j} \le 1, \qquad j = 1,...,n, \quad p = 1,...,n, \tag{6.2}$$

$$\theta_{pj} = \frac{u_p^* y_j}{v_p^* x_j}, \qquad j = 1,...,n, \quad p = 1,...,n, \tag{6.3}$$

$$\bar{\theta}_j = \frac{1}{n} \sum_{p=1}^{n} \theta_{pj}, \qquad j = 1,...,n, \tag{6.4}$$

$$\bar{\theta}_j - \bar{\theta}_o \le M(1 - I_j), \qquad j = 1,\dots,n, j \ne o,$$
$$u_p^* \ge 0_s, v_p^* \ge 0_m, \qquad p = 1,\dots,n, I_j \in \{0,1\}, \quad j = 1,\dots,n, j \ne o, \tag{6.5}$$

where θ_p^* is the efficiency score of DMUP provided by *Model (2).* $I_j, j \ne o$ are binary variables that, at optimum, state DMU_j outperforms DMU_o or not. Also, M is a sufficient large positive number. In *Model (6)*, we aim to minimize the number of better DMUs. In the optimal solution of *Model (6)*, $I_j = 0$ indicates that DMU_j is better than DMU_p if and only if $\bar{E}_j > \bar{E}_o$, while $\bar{E}_j \le \bar{E}_o$ is necessarily associated with $I_j = 1$.

Therefore, *Model (6)* computes the maximum number of units whose cross-efficiency scores are lower than or equal to DMU_p (excluding DMU_p). Thus, the best ranking of DMU_p can be easily obtained using *Eq. (5).* Alcaraz et al. [1] proposed a model similar to *Model (6)* to determine the worst ranking of DMU_o. In this vein, the following *Constraint (7)* is put instead of *Constraint (6.5)* in *Model (6).*

$$\bar{\theta}_o - \bar{\theta}_j \le M(1 - I_j), \qquad j = 1,\dots,n, j \ne o. \tag{7}$$

Therefore, the worst ranking of the DMU_o is obtained by:

$$r_o^w = BE_o^* + 1, \tag{8}$$

where BE_o^* is the optimal solution of *Model (6)*, while *Constraint (7)* is located instead of *Constraint (6.5)* in *Model (6).* One of the weaknesses of both models proposed by Alcaraz et al. [1] is their non-linearity; that is, they are Mixed-Integer Linear Programs (MILPs) built upon the set of feasible cross-efficiency weights derived from the DEA models. To solve this problem, we propose a new method for converting non-linear models [1] into LP models.

3 | Proposed Method

The problems with the methods suggested by Alcaraz et al. [1] are the nonlinearity and the integer programming. To deal with these problems, we use goal programming to convert the MILP model into an LP model. In this article, for the first time, a Goal-programming problem is proposed to determine the optimal ranking of the DMU under evaluation, which is applied instead of *Model (6).*

Now, we propose *Model (9)* based on goal programming to find the best rank for DMU_o.

$$\min \quad WE_o = \sum_{j=1}^{n} d_j^+, \tag{9}$$

$$\text{s. t.} \quad \frac{u_p^* y_p}{v_p^* x_p} = \theta_p^*, \qquad p = 1,\dots,n, \tag{9.1}$$

$$\frac{u_p^* y_j}{v_p^* x_j} \le 1, \qquad j = 1,\dots,n, \quad p = 1,\dots,n, \tag{9.2}$$

$$\theta_{pj} = \frac{u_p^* y_j}{v_p^* x_j} \qquad j = 1,\dots,n, \quad p = 1,\dots,n \tag{9.3}$$

$$\bar{\theta}_j = \frac{1}{n}\sum_{p=1}^{n} \theta_{pj} \qquad j = 1,\dots,n, \tag{9.4}$$

$$\bar{\theta}_j - \bar{\theta}_o + d_j^- - d_j^+ = 0, \qquad j = 1,\dots,n, j \ne o,$$
$$u_p^* \ge 0_s, v_p^* \ge 0_m, d_j^- \ge 0, d_j^+ \ge 0, \qquad p = 1,\dots,n, j = 1,\dots,n, \tag{9.5}$$

where WE_p^* is the maximum number of units whose cross-efficiency scores in the above-mentioned goal-programming problem are lower than or equal to DMU_o. Also, θ_p^* is the efficiency score of DMUp attained by *Model (2)*. The objective function of *Model (9)* minimizes unfavorable deviation. In *Model (9),* if $d_j^- > 0$ and $d_j^+ = 0$ then $\bar{\theta}_j < \bar{\theta}_o$, it satisfies that DMU_j performs worse than DMU_o under evaluation.

After solving *Model (9)*, the worst ranking of the DMU under assessment can be computed by *Relation (8)*. Now, the goal-programming *Model (10)* is expressed as follows, which is suggested to find the worst ranking of DMU_o.

$$\min \quad BE_o = \sum_{j=1}^{n} d_j^-, \tag{10}$$

$$\text{s. t.} \quad \frac{u_p^* y_p}{v_p^* x_p} = \theta_p^*, \qquad p = 1,...,n, \tag{10.1}$$

$$\frac{u_p^* y_j}{v_p^* x_j} \le 1, \qquad j = 1,...,n, \quad p = 1,...,n, \tag{10.2}$$

$$\theta_{pj} = \frac{u_p^* y_j}{v_p^* x_j}, \qquad j = 1,...,n, \quad p = 1,...,n, \tag{10.3}$$

$$\bar{\theta}_j = \frac{1}{n}\sum_{p=1}^{n} \theta_{pj}, \quad j = 1,...,n, \tag{10.4}$$

$$\begin{aligned} &\bar{\theta}_j - \bar{\theta}_o + d_j^- - d_j^+ = 0, \qquad j = 1,...,n, j \ne o, \\ &u_p^* \ge 0_s,\ v_p^* \ge 0_m,\ d_j^- \ge 0, d_j^+ \ge 0, \qquad p = 1,...,n,\ j = 1,...,n, \end{aligned} \tag{10.5}$$

where BE_p^* is the maximum number of units whose efficiency scores in the above-mentioned goal-programming are higher than or equal to DMU_o. This problem is the same as *Model (9)*, but in the objective function of *Model (10)*, we have $\sum_{j=1}^{n} d_j^-$ instead of $\sum_{j=1}^{n} d_j^+$. Finding the maximum number of units whose cross-efficiency is higher than or equal to DMUo is the aim of *Model (10)*. In *Model (10)*, if $d_j^- = 0$ and $d_j^+ > 0$ then $\bar{E}_j > \bar{E}_o$, it means that DMU_j performs better than DMU_o. Therefore, the worst ranking of DMU_o is attained by solving *Model (10)* and *Relation (5)*.

4 | Numerical Example

The above-mentioned goal-programming problem is applied to the data mentioned in the study, in which six nursing homes are evaluated with two inputs and two outputs.

Now, we apply the proposed models to numerical examples from Alcaraz et al. [1] and compare the results with previous ones. Consider the example of Alcaraz et al. [1] where six nursing homes are evaluated with two inputs (X_1 and X_2) and two outputs (Y_1 and Y_2). *Table 1* shows the data along with the efficiency scores obtained from the CCR model (*Model (2)*).

Table 1. Data example [1].

DMU	X_1	X_2	Y_1	Y_2	DEA Score
1	1.5	0.2	1.4	0.35	1
2	4	0.7	1.4	2.1	1
3	3.2	1.2	4.2	1.05	1
4	5.2	2	2.8	4.2	1
5	3.5	1.2	1.9	2.5	0.9775
6	3.2	0.7	1.4	1.5	0.8674

Obviously, four DMUs out of six are CCR-efficient with a score of 1. DMU_5 and DMU_6 are inefficient.

Table 2 reports the ranking ranges for each unit, obtained by solving Model (6) and Constraints (5) and (8), as proposed by [1]. Results are reported in *Table 2*.

Table 2. Ranking ranges of six DMUs [1].

DMU	Best Ranking (Model (6))	Worse Ranking (Model (7))
1	1	6
2	1	5
3	1	6
4	1	4
5	2	5
6	4	6

Now, we consider DMU_5 to determine its worst and best ranks. To achieve this goal, *Models (9)* and *(10)* are solved. *Table 3* presents the deviation value from DMU_5 by utilizing *Model (10)* and *Relation (8)*.

Table 3. Worst ranking of DMU_5 via solving Model (10).

d_1^-	d_2^-	d_3^-	d_4^-	d_6^-	d_1^+	d_2^+	d_3^+	d_4^+	d_6^+
0	0	0	0.001536214	0.02305003	0.2750802	0.1062967	0	0	0

Similarly, by considering *Model (9)* and *Relation (5)*, the best ranking of DMU_5 is determined. *Table 4* represents the deviation amount of DMU_5, by considering the result of *Table 3* $d_1^{-*} = d_2^{-*} = d_2^{-*} = 0$. Therefore, the amount of the objective function *Model (10),* i.e., $\sum_{j=1}^{n} d_j^-$ will be equal to three. The worst rank of DMU_5 by considering Phrase (8) is obtained as follows:

$$r_5^w = BE_o^* = \sum_{j=1}^{n} d_j^{-*} + 1 = 3 + 1 = 4.$$

The worst rank for DMU_5 among the six DMUs is 4. Similarly, regarding *Model (9)* and the result of *Table 4*, we have $d_1^{+*} = d_2^{+*} = d_3^{+*} = d_6^{+*} = 0$ then the value of the objective function *Model (10),* i.e, $\sum_{j=1}^{n} d_j^+$ is equal to four. Finally, the best rank for DMU_5 upon *Relation (5)* is obtained as follows:

$$r_5^b = n - WE_o^* = 6 - 4 = 2.$$

The best rank for DMU_5 among the six NITs is 2. Finally, applying the same procedure to the other DMUs, ranking ranges are computed, as shown in *Table 5*.

Table5. Ranking ranges of all DMUs by using the proposed method.

DMU	Best Ranking	Worse Ranking
1	1	6
2	1	6
3	1	6
4	1	6
5	2	4
6	5	6

From *Table 5*, we see that each unit can almost accept every ranking when evaluated using cross-efficiency. DMU_1 and DMU_3 can be ranked first or last according to the DEA weights. DMU_6 is the worst performer, since it always accepts the lowest rankings.

5 | Conclusion

DEA combined with cross-efficiency evaluation enables a more precise assessment of performance and a fair ranking of DMUs. The cross-efficiency method addresses the limitations of traditional DEA by introducing peer-assessment perspectives, yielding more consistent and discriminating outcomes. In the cross-efficiency evaluation approach within DEA, each DMU is first assessed using its own optimal input-output weights, and then re-evaluated using the weights of other DMUs.

Ranking DMUs is a significant issue for organizations. Many studies have applied the DEA method to rank units. However, because DEA model weights are not unique, it is sometimes not possible to assign a unique rank to each unit. To address this problem, some models have been developed based on DEA and cross-efficiency evaluation. In this context, Alcaraz et al. [1] introduced a new concept, called ranking ranges, which was achieved by solving two non-LP problems. In this paper, building on their idea and using the goal programming method, two LP problems are proposed that do not suffer from the issues associated with integer programming and provide a procedure to obtain the worst- and best-ranked rankings for each DMU. The advantage of our proposed method compared to other methods is its linearity. This makes the problem easier to solve and reduces system and time costs.

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